Generalized Quantifiers, Determiners and Monotonicity

In class we discussed the denotation of sentences as in A-C and expressed it using a GQ denotation for the NP. The VP denotation is required to be a member of the GQ. As a result the sentences express relations between the set denotation of the noun and the set denotation of the VP:

A. Every man ran - \( \text{run} \in \{ B \subseteq E : \text{man} \subseteq B \} \leftrightarrow \text{man} \subseteq \text{ran} \) - the set of runners is an element of the set of sets that contain the set of men ↔ the set of men is contained in the set of runners

B. Some man ran - \( \text{run} \in \{ B \subseteq E : \text{man} \cap B \neq \emptyset \} \leftrightarrow \text{man} \cap \text{ran} \neq \emptyset \) - ... ↔ the set of men has a non-empty intersection with the set of runners

C. No man ran - \( \text{run} \in \{ B \subseteq E : \text{man} \cap B = \emptyset \} \leftrightarrow \text{man} \cap \text{ran} = \emptyset \) - ... ↔ the set of men has an empty intersection with the set of runners

Answer the following questions:

Expressing GQs

a. Replace the “…”s in (B) and (C) by full sentences that reflect the GQ analysis, following the example in (A).

b. For the following sentences, write down an analysis similar to the analyses in A-C:
   i. At most one man ran.
   ii. Between three and eleven men ran.
   iii. All the men except John ran.
   iv. No man except John ran.

Note: for the sake of this question you may assume that the denotation of the plural noun \textit{men} is \textit{man}, like the singular noun \textit{man}. 
Answers:

Generalized Quantifiers

a. Filling the “…”
   i. The set of runners is an element of the set of sets that have a non-
      empty intersection with the set of man.
   ii. The set of runners is an element of the set of sets that have an
      empty intersection with the set of man.

b. Analysis of GQs:

1. At most one man \(- run \in \{B \subseteq E : |man \cap B| \leq 1\} \leftrightarrow |man \cap ran| \leq 1\) - the set of runners is an element of the set of sets such that there’s one
   or less entities in their intersection with the set of man \(\leftrightarrow\) there is one or
   less entities in the intersection of the set of man and the set of runners.

2. Between three and eleven men ran \(- run \in \{B \subseteq E : 3 \leq |man \cap B| \leq 11\} \leftrightarrow 3 \leq |man \cap ran| \leq 11\) - the set of runners is an element of the
   set of sets such that there are at least 3 and at most 12 entities in their
   intersection with the set of man \(\leftrightarrow\) there are at least 3 and at most 12
   entities in the intersection of the set of man and the set of runners.

3. All the men except John ran \(- run \in \{B \subseteq E : man \setminus \{john\} \subseteq B \land \{john\} \not\subseteq B\} \leftrightarrow man \setminus \{john\} \subseteq run \land \{john\} \not\subseteq run\) - the set of
   runners is an element of the set of sets that contain all men excluding John,
   and John is not a member in them \(\leftrightarrow\) the set of men excluding john is
   contained in the set of runners, and John is not in the set of runners.

   An alternative solution:
   All the men except John ran \(- run \in \{B \subseteq E : man \cap B = man \setminus \{john\}\} \leftrightarrow man \cap runners = man \setminus \{john\}\) - the set of
   runners is an element of the set of sets that their intersection with the
   set of man equals the set of man excluding John \(\leftrightarrow\) the set of men
   intersected with the set of runners equals the set of man excluding
   John.

4. No man except John ran. \(- run \in \{B \subseteq E : man \cap B = \{john\}\} \leftrightarrow\)
   \(man \cap runners = \{john\}\) - the set of runners is an element of the set of
   sets that their intersection with the set of man equals to the set that
   contains only John. \(\leftrightarrow\) the set of men intersected with the set of runners
   equals the set that contains only John.
Determiners

a. Write the semantic type of each word in sentence (A). Hint: we now assume type \((et)t\) for NPs. You should analyze the required type of the determiner in the noun phrase every man, so that the NP ends up having the \((et)t\) type.

b. Describe the functions in the domain \((et)((et)t)\) in informal mathematical language (the functions that send every ... to ...)

c. These functions characterize binary relations between functions of type X. What is X?

d. Following these observations conclude: these functions characterize binary relations between sets of ____
Answers:

a. Semantic types:
   i. man: et
   ii. ran: et
   iii. every: (et)((et)t)

b. Functions that take a function from entities to truth values as an argument and return a function that takes a function from entities to truth values as an argument and returns a truth value.

c. X is et

d. Entities
Monotonicity

A generalized quantifier \( Q \subseteq \wp(E) \) is:
- \( \text{MON}^\uparrow \) iff for all \( A \subseteq B \subseteq E \): if \( A \in Q \) then \( B \in Q \)
- \( \text{MON}^\downarrow \) iff for all \( A \subseteq B \subseteq E \): if \( B \in Q \) then \( A \in Q \)
- Non-monotone otherwise.

Consider the following sentences

(1) Every man but no woman ran
(2) Every man but no woman ran quickly

Q1. Show that \textit{every man but no woman} is non-monotonic in the following way:

1. Give set denotations to \textit{man}, \textit{woman}, \textit{ran} and \textit{quickly(ran)} in which (1) denotes 1 and (2) denotes 0
2. Give set denotations to \textit{man}, \textit{woman}, \textit{ran} and \textit{quickly(ran)} in which (1) denotes 0 and (2) denotes 1

Q2. Write down the *denotation* of the noun phrase \textit{every man but no woman} as a set intersection of the denotations for the noun phrases \textit{every man} and \textit{no woman}

Q3. Specify the sets intersected in the QC given a domain of entities and set denotations as follows:
\( E = \{ \text{john, bill, mary, sue, mitsy, fritsy} \} \)
\( \text{man} = \{ \text{john, bill} \} \)
\( \text{woman} = \{ \text{mary, sue} \} \)
Answers
Q1

Showing that *every man but no woman* is a non-monotonic GQ:

a. Set denotations in which (1) denotes 1 and (2) denotes 0:
   man: \{guy, louis, ray\}
   woman: \{christin, janet, isabel\}
   ran: \{guy, louis, ray\}
   quickly(ran): \{ray\}

   Note: this shows that the GQ is not upward monotonic.

b. Set denotations in which (1) denotes 0 and (2) denotes 1:
   man: \{guy, louis, ray\}
   woman: \{christin, janet, isabel\}
   ran: \{guy, louis, ray, janet\}
   quickly(ran): \{guy, louis, ray\}

   Note: this shows that the GQ is not downward monotonic.
A visual illustration of a model in which:

a. *Every man but no woman ran* denotes 1
b. *Every man but no woman ran quickly* denotes 0
Q2

*every man* denotes: \( \{ B \subseteq E : \text{man} \subseteq B \} \)

*no woman* denotes: \( \{ B \subseteq E : \text{woman} \cap B = \emptyset \} \)

Their intersection denotes: \( \{ B \subseteq E : \text{man} \subseteq B \} \cap \{ B \subseteq E : \text{woman} \cap B = \emptyset \} \)

Q3

In the given model,

*every man* denotes: \{ \{john, bill, mary, sue, mitsy, fritsy\},
\{john, bill, mary, sue, mitsy\},
\{john, bill, mary, sue, fritsy\},
\{john, bill, mary, mitsy, fritsy\},
\{john, bill, sue, mitsy, fritsy\},
\{john, bill, mary, sue\},
\{john, bill, mary, mitsy\},
\{john, bill, sue, mitsy\},
\{john, bill, mary, fritsy\},
\{john, bill, sue, fritsy\},
\{john, bill, mitsy, fritsy\},
\{john, bill, sue\},
\{john, bill, mitsy\},
\{john, bill, fritsy\},
\{john, bill}\} 

*no woman* denotes: \{ \{john, bill, mitsy, fritsy\},
\{john, bill, mitsy\},
\{john, bill, fritsy\},
\{john, mitsy, fritsy\},
\{john, fritsy\},
\{bill, mitsy\},
\{bill, fritsy\},
\{mitsy, fritsy\},
\{john,\}, \{bill,\}, \{mitsy,\}, \{fritsy,\}, \emptyset \} 

The intersection of *every man* and *no woman* is:

\{ \{john, bill, mary, sue, mitsy, fritsy\},\{john, bill, mary, sue, mitsy\},
\{john, bill, mary, sue, fritsy\},\{john, bill, mary, mitsy, fritsy\},
\{john, bill, sue, mitsy, fritsy\},\{john, bill, mary\},
\{john, bill, mitsy\},\{john, bill, fritsy\},
\{john, bill, mitsy\},\{john, bill, sue\},
\{john, bill, mary, fritsy\},\{john, bill, sue, mitsy\},
\{john, bill, mary, fritsy\},\{john, bill, sue, fritsy\},
\{john, bill, mitsy, fritsy\},\{john, bill, mitsy\},\{john, bill, sue\},
\{john, bill, mitsy\},\{john, bill, fritsy\},\{john, bill, mitsy\},\{john, bill, fritsy\},
\{john, bill, mitsy\},\{john, bill, fritsy\},\{john, bill, mitsy\},\{john, bill\},\{john, mitsy\},\{john, fritsy\},
\{bill, mitsy\},\{bill, fritsy\},\{mitsy, fritsy\},\{john,\}, \{bill,\}, \{mitsy,\}, \{fritsy,\}, \emptyset \} = \{ \{john, bill, mitsy, fritsy\}, \{john, bill, mitsy\}, \{john, bill, fritsy\}, \{john, bill\}\}