Types and Model Structure (cont.)

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Foundations Semantics & Pragmatics

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Our model of entailment

1- Functions – useful as denotations
2- Types and domains – organize models
3- Models – and their interaction with syntax
Functions again
Given two sets $A$ and $B$, a function from $A$ to $B$ is a rule or procedure that "inputs" elements of $A$ and "outputs" elements of $B$.

Notation:

$$f : A \rightarrow B$$

Every element $a \in A$ gets "sent to" some element of $B$ which is called $f(a)$.

Functions are also called mappings.

$A$ is called the domain of $f$, and $B$ the codomain or range.
One-to-one and onto

\[ f : A \rightarrow B \] is one-to-one if different inputs go to different outputs.

\[ f : A \rightarrow B \] maps onto \( B \) if every element of \( B \) is the output of some element of \( A \), possibly more than one.

(Often people say “\( f \) is onto”, using a preposition as an adjective. Clearly they have no regard for traditional grammar!)

Fancier names: injective for one-to-one. surjective for “maps onto its codomain.”
Sets having the same size

We say that two sets $A$ and $B$ have the same size if there is a function $f : A \rightarrow B$ which is one-to-one and maps onto $B$.

These conditions can be expressed more formally:

If $a \neq a'$ in $A$, then $f(a) \neq f(a')$.

For every $b$, there is some $a \in A$ such that $f(a) = b$. 
Example

Here are two sets:

- Numbers = \{0, 1, 2, 3, \ldots\}
- Even = \{0, 2, 4, 6, \ldots\}

It might be surprising to think that these two sets have the same size, but this is what the definitions tell us.

To be sure, we need to find some function

\[ f : \text{Numbers} \rightarrow \text{Even} \]

which is one-to-one and onto.

We can take

\[
\begin{align*}
  f(0) &= 0 \\
  f(1) &= 2 \\
  f(2) &= 4 \\
  \vdots & \quad \vdots \\
  f(n) &= 2n \\
  \vdots & \quad \vdots
\end{align*}
\]
Sets of functions

For any two sets $A$ and $B$, the set of all functions from $A$ to $B$ is denoted

$$B^A$$

alternatively, $A \rightarrow B$

Example:

$$\{a, b, c\}^{\{1,2\}} = \text{the following functions:}$$

$$1 \mapsto a \quad 2 \mapsto a$$
$$1 \mapsto a \quad 2 \mapsto b$$
$$1 \mapsto a \quad 2 \mapsto c$$
$$1 \mapsto b \quad 2 \mapsto a$$
$$1 \mapsto b \quad 2 \mapsto b$$
$$1 \mapsto b \quad 2 \mapsto c$$
$$1 \mapsto c \quad 2 \mapsto a$$
$$1 \mapsto c \quad 2 \mapsto b$$
$$1 \mapsto c \quad 2 \mapsto c$$
Types and Domains
A type is a label for part of a model that is called a domain.

**Basic** types and domains:
- \( e : D_e \) - arbitrary - of entities
- \( t : D_t = \{0,1\} \) - of truth-values

**Complex** types and domains: defined inductively from basic types and domains.
Example

\[ E = D_e = \text{the set of entities } \{t,j,m\} \]

\[[\text{thin}]] = T = \{t,j\}

We can also define \( T \) as a function from \( D_e \) to \( D_t \):

\[
\begin{align*}
    t &\rightarrow 1 \\
    j &\rightarrow 1 \\
    m &\rightarrow 0
\end{align*}
\]

This function \textbf{characterizes} \( T \) in \( E = D_e \).

\( D_{et} \) of the complex type \textbf{et} is the domain of such functions.
## Characteristic functions over \{t,j,m\}

<table>
<thead>
<tr>
<th>Subset of (D_e)</th>
<th>Function in (D_{et})</th>
</tr>
</thead>
</table>
| \(\emptyset\)   | \(f_1: t \mapsto 0\)  
                      | \(j \mapsto 0\)  
                      | \(m \mapsto 0\) |
| \{m\}           | \(f_2: t \mapsto 0\)  
                      | \(j \mapsto 0\)  
                      | \(m \mapsto 1\) |
| \{j\}           | \(f_3: t \mapsto 0\)  
                      | \(j \mapsto 1\)  
                      | \(m \mapsto 0\) |
| \{j, m\}        | \(f_4: t \mapsto 0\)  
                      | \(j \mapsto 1\)  
                      | \(m \mapsto 1\) |
| \{t\}           | \(f_5: t \mapsto 1\)  
                      | \(j \mapsto 0\)  
                      | \(m \mapsto 0\) |
| \{t, m\}        | \(f_6: t \mapsto 1\)  
                      | \(j \mapsto 0\)  
                      | \(m \mapsto 1\) |
| \{t, j\}        | \(f_7: t \mapsto 1\)  
                      | \(j \mapsto 1\)  
                      | \(m \mapsto 0\) |
| \{t, j, m\}     | \(f_8: t \mapsto 1\)  
                      | \(j \mapsto 1\)  
                      | \(m \mapsto 1\) |

**Table 2.1:** Subsets of \(D_e\) and their characteristic functions in \(D_{et}\)
Characteristic Functions

Let $X$ be any set.

Every subset $A \subseteq X$ gives us a function $f_A : X \to \{0, 1\}$ called the characteristic function of $A$.

It is defined as follows:

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

**Example:** $X = \{a, b, c, d\}$, $A = \{a, c\}$

Here are some examples of how this function $f_A$ works:

- $f_A(a) = 1$.
- $f_A(b) = 0$.
- $f_A(c) = 1$.
- $f_A(d) = 0$. 
Definition 1. The set of types over the basic types $e$ and $t$ is the smallest set $\mathcal{T}$ that satisfies:

(i) $\{e, t\} \subseteq \mathcal{T}$

(ii) If $\tau$ and $\sigma$ are types in $\mathcal{T}$ then $(\tau \sigma)$ is also a type in $\mathcal{T}$.

\[
e, t, \\
ee, tt, et, te, \\
e(ee), e(tt), e(et), e(te), t(ee), t(tt), t(et), t(te), \\
(ee)e, (tt)e, (et)e, (te)e, (ee)t, (tt)t, (et)t, (te)t, \\
(ee)(ee), (ee)(tt), (ee)(et), (ee)(te), (tt)(ee), (tt)(tt), (tt)(et), (tt)(te)
\]

Definition 2. For all types $\tau$ and $\sigma$ in $\mathcal{T}$, the domain $D_{\tau \sigma}$ of the type $(\tau \sigma)$ is the set $D_{\sigma}^{D_{\tau}}$ – the functions from $D_{\tau}$ to $D_{\sigma}$. 

Intransitive verbs

Tina smiled.

\[ \text{smile}_{et}(\text{tina}_e) \]
Function Application

From types $e$ and $et$, FA gives $t$ (as we have seen above).
From types $(e(et))(et)$ and $e(et)$, FA gives $et$.
Types $(e(et))(et)$ and $et$ cannot combine using FA: neither of these types is a prefix of the other.

Function Application (FA):

\[(ab) + a = b\]
\[f + x = f(x)\]
Intransitive and Transitive verbs

Tina smiled.
Tina [praised Mary].

\( \text{smile}_{et}(tina_e) \)

\( (\text{praise}_{e(et)}(mary_e))(tina_e) \)

or

\( \text{praise}(mary)(tina) \)
“Curried” Relations

\[ U = \{ (t, m), (m, t), (m, j), (m, m) \} \]

\[ f_U : \begin{array}{llll}
  t & \mapsto & [t \mapsto 0 & j \mapsto 0 & m \mapsto 1] \\
  j & \mapsto & [t \mapsto 0 & j \mapsto 0 & m \mapsto 1] \\
  m & \mapsto & [t \mapsto 1 & j \mapsto 0 & m \mapsto 1] 
\end{array} \]

- \( f_U \) maps the entity \( t \) to the function characterizing the set \( \{ m \} \).
- \( f_U \) maps the entity \( j \) to the function characterizing the same set, \( \{ m \} \).
- \( f_U \) maps the entity \( m \) to the function characterizing the set \( \{ t, m \} \).

When the function \( f_U \) is the denotation of the verb \textit{praise}, and the entities \( t, j \) and \( m \) are the denotations of the respective names, this is the situation where:

- Mary is the only one who praised Tina.
- Mary is the only one who praised John.
- Tina and Mary, but not John, praised Mary.
Currying

F: $(M \times W) \rightarrow [0, 1]$
F gives any pair of man and woman $(m, w)$ a score $F(m, w)$ indicating matching

G: $M \rightarrow (W \rightarrow [0, 1])$
G gives any man $m$ a function $G(m)$ mapping any woman $w$ to a score $(G(m))(w)$.

Thus, we can define: $(G(m))(w) = F(m, w)$
We say that G is the Curried version of F, and that F is the deCurried version of G.
Use of Currying for compositional interpretation of binary structures

A.

Tina
  praised
   Mary

B.

praise(mary)(tina) : t

  tina : e
  praise(mary) : et

    praise : e(et)
    mary : e
Modifiers

Mary [walked quickly]

Mary walked
Non-arbitrary Denotations: IS

For every function $f$ in $D_{et}$: $\text{is}(f) = f$.

A. Tina
   is
   tall

B. $(\text{is}(\text{tall}))(\text{tina}) : t$
   tina : $e$
   is(tall) : $et$
   is : $(et)(et)$
   tall : $et$

Alternative structures – alternative types?
Non-arbitrary Denotations: AND

For every two functions $f_A$ and $f_B$ in $D_e$, characterizing the subsets $A$ and $B$ of $D_e$: $(\text{AND}(f_A))(f_B)$ is defined as the function $f_{A \cap B}$, characterizing the intersection of $A$ and $B$.

**Explain:**
Tina is tall and thin $\implies$ Tina is thin

**Types?**
## In General

**Types of the form:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Syntactic/semantic role</th>
</tr>
</thead>
<tbody>
<tr>
<td>at</td>
<td>?</td>
</tr>
<tr>
<td>a(at)</td>
<td>?</td>
</tr>
<tr>
<td>a(a(at))</td>
<td>?</td>
</tr>
<tr>
<td>...</td>
<td>?</td>
</tr>
<tr>
<td>aa</td>
<td>?</td>
</tr>
<tr>
<td>a(aa)</td>
<td>?</td>
</tr>
<tr>
<td>a(a(aa))</td>
<td>?</td>
</tr>
<tr>
<td>...</td>
<td>?</td>
</tr>
</tbody>
</table>
### In General

**Types of the form**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$at$</td>
<td>1-place predicates</td>
<td>smile</td>
</tr>
<tr>
<td>$a(at)$</td>
<td>2-place predicates</td>
<td>praise</td>
</tr>
<tr>
<td>$a(a(at))$</td>
<td>3-place predicates</td>
<td>send</td>
</tr>
<tr>
<td>…</td>
<td>n-place predicates</td>
<td></td>
</tr>
<tr>
<td>$aa$</td>
<td>modifiers</td>
<td>quickly</td>
</tr>
<tr>
<td></td>
<td>(1-place coordinators)</td>
<td></td>
</tr>
<tr>
<td>$a(aa)$</td>
<td>2-place coordinators</td>
<td>M and J</td>
</tr>
<tr>
<td>$a(a(aa))$</td>
<td>3-place coordinators</td>
<td>M, J and S</td>
</tr>
<tr>
<td>…</td>
<td>n-place coordinators</td>
<td></td>
</tr>
</tbody>
</table>
What would be the type of IS with the following (infelicitous) structure?

[Tina is] tall

What denotation would we assume for IS?
Some examples to think about

Mary [walked [in Utrecht]]
[Walk -ing] [is fun]
[[Walk -ing] [in Utrecht]] [is fun]
[The man] smiled
[The [tall man]] smiled
[If [you smile]] [you win]
There [is [trouble [in Paradise]]]
I [[love it] [when [you smile]]]