Foundations of Semantics and Pragmatics

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www.hum.uu.nl/medewerkers/h.devries1/fosp1213.html

MA Linguistics, Utrecht University
Fall 2012/3
Course Aims

- Formal approach to natural language meaning
- Review of selected research domains
- Academic skills
Topics

Part 1 – formal semantics:
- Background
- Set theory
- Types and models
- Lambda’s
- Quantification

Part 2 – selected topics:
- Pragmatics
- Degree semantics
- Plurality
- Events
- Experimental pragmatics
Course Grade

40% Midterm exam (part 1)
  obligatory passing grade

20% Homework assignments
  10%: 5 hw’s part 1: obligatory passing grade
  10%: 2 hw’s part 2: fully graded

20% Class presentation

20% Extended abstract of presentation (5pp.)
Course Material

Part 1:

- Partee, ter Meulen and Wall: *Mathematical Methods in Linguistics* Ch.1-3
- Partee’s *Memories*
- Lecture notes Yoad + textbook drafts
- Gamut 1+2

Part 2:

- Will appear on website
Please note

Course locations:
- Tuesday: 09.00-10.45 D23 206
- Thursday: 15.15-17.00 ADD 001

Please subscribe to mailing list through the course site.

Communication with teachers preferably in person after class.
Let’s go!

1- Historical background
2- Meaning and form
3- Set theory
From Language to Logic

Gottlob Frege
(1848-1925)
Prove It!

David Hilbert
(1862-1943)
A Blow to Formalism

Kurt Gödel
(1906-1978)
Model it!

Alfred Tarski
(1902-1983)
Meanwhile in Cognitive Science

“It seems clear, then that undeniable, though only imperfect correspondences hold between formal and semantic features in language.”

(Syntactic Structures, 1957)

“...Chomsky would say [that] the semantic purposes do not determine the form of the syntax or even influence it in any significant way.”

(Chomsky's Revolution in Linguistics, John R. Searle, 1972)
Towards a Synthesis

"There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of language within a single natural and mathematically precise theory. On this point I differ from a number of philosophers, but agree, I believe, with Chomsky and his associates."

(Universal Grammar, 1970)

Richard Montague (1930-1971)
The Key to Montague’s Program

Frege’s Principle of Compositionality
The meaning of a compound expression is a function of the meanings of its parts, and the ways they combine with each other.
Ambiguous Expressions

I saw the man with the telescope
Syntactic Ambiguity

I saw the man with the telescope
I saw the man with the telescope
Syntactic-Semantic Ambiguity

I saw the man with the telescope
I saw the man with the telescope
Mentalist vs. Linguistic Meaning Relations

(1)  a. What is common to the objects that people categorize as being red?
    b. How do people react when they are addressed with the request please pick a blue card from this pack?
    c. What emotions are invoked by expressions like my sweetheart, my grandmother or my boss?

(2)  a. How do speakers identify relations between pairs of words like red-color, dog-animal and chair-furniture?
    b. What are the relations between the use of the imperative sentence please pick a blue card from this pack and the use of the similar sentence please pick a card from this pack?
    c. How are the descriptions my grandmother and my only living grandmother related to each other in language use?

(3)  Red is a color / ?Red is an animal
(4)  The color red annoys me / ?The animal red annoys me
(5)  Every red thing has a color / ?Every red thing has an animal
Entailment

(6) Tina is tall and thin.

From this premise, any speaker of English is able to draw the conclusion in sentence (7).

(7) Tina is thin.

We say that sentence (6) entails (7), and denote it (6)⇒(7). In this entailment, we call sentence (6) the premise, or antecedent. Sentence (7) is called the conclusion, or consequent.

(8) a. A dog entered the room ⇒ An animal entered the room
b. John picked a blue card from this pack ⇒ John picked a card from this pack
c. I met my only living grandmother yesterday ⇒ I met my grandmother yesterday

(9) Tina is a bird.

(10) Tina can fly.

(11) Tina is a bird, but she cannot fly, because... (she is too young to fly, a penguin, an ostrich, etc.)

(12) #Tina is tall and thin, but she is not thin, because...

**Entailment** is the indefeasible relation, denoted $S_1⇒S_2$, between a premise $S_1$ and a valid conclusion $S_2$ expressed as natural language sentences.
Let \( \text{exp} \) be a language expression, and let \( M \) be a model. We write \( [[\text{exp}]]^M \) when referring to the denotation of \( \text{exp} \) in the model \( M \).

A semantic theory \( T \) is said to satisfy the **truth-conditionality criterion** (TCC) if for all sentences \( S_1 \) and \( S_2 \), the following two conditions are equivalent:

I. Sentence \( S_1 \) intuitively entails sentence \( S_2 \).

II. For all models \( M \) in \( T \): \( [[S_1]]^M \leq [[S_2]]^M \).

<table>
<thead>
<tr>
<th>( x = 0 )</th>
<th>( y = 0 )</th>
<th>( y = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>no</td>
<td>yes</td>
<td></td>
</tr>
</tbody>
</table>

\[ S_1 \Rightarrow S_2 \]
Assumptions about our models

1. In every model $M$, in addition to the two truth-values, we also have an arbitrary non-empty set $E_M$ of entities in $M$, which contains the simplest objects in this model.

2. In any model $M$, the proper name $Tina$ denotes an arbitrary entity in $E_M$.

3. In any model $M$, the adjectives $tall$ and $thin$ denote arbitrary sets of entities in $E_M$.

$$\text{IS}(x, A) = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{if } x \notin A
\end{cases}$$

$\text{AND}(A, B) = A \cap B$ = the set of all members of $E$ that are both in $A$ and in $B$

Thus:

$$[[Tina \text{ is thin}]]^M = \text{IS}(tina, \text{thin})$$

$$[[Tina \text{ is tall and thin}]]^M = \text{IS}(tina, \text{AND}(\text{tall, thin}))$$

Convention:

Let blik be a word in a language. When the denotation $[[\text{blik}]]^M$ of blik is arbitrary, we mark it blik, and when it is constant across models we mark it BLIK. In both notations the model $M$ is implicit.
<table>
<thead>
<tr>
<th>Expression</th>
<th>Cat.</th>
<th>Type</th>
<th>Abstract denotation</th>
<th>Denotations in example models with $E = {a, b, c, d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Tina$</td>
<td>PN</td>
<td>entity</td>
<td>$tina$</td>
<td>$a$</td>
</tr>
<tr>
<td>$tall$</td>
<td>A</td>
<td>set of entities</td>
<td>$tall$</td>
<td>${b, c}$</td>
</tr>
<tr>
<td>$thin$</td>
<td>A</td>
<td>set of entities</td>
<td>$thin$</td>
<td>${a, b, c}$</td>
</tr>
<tr>
<td>$tall$ and $thin$</td>
<td>AP</td>
<td>set of entities</td>
<td>$\text{AND}(tall, thin)$</td>
<td>${a, b, c}$</td>
</tr>
<tr>
<td>$Tina$ is $thin$</td>
<td>S</td>
<td>truth-value</td>
<td>$\text{IS}(tina, thin)$</td>
<td>${b, c}$</td>
</tr>
<tr>
<td>$Tina$ is $tall$ and $thin$</td>
<td>S</td>
<td>truth-value</td>
<td>$\text{IS}(tina, \text{AND}(tall, thin))$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

**Categories:** PN = proper name; A = adjective; AP = adjective phrase; S = sentence

**Table 2.2:** Denotations for expressions in the entailment $(6) \Rightarrow (7)$
Compositionality

(15) a. All pianists are composers, and Tina is a pianist.
   b. All composers are pianists, and Tina is a pianist.

**Compositionality**: The denotation of a complex expression is determined by the denotations of its immediate parts and the ways they combine with each other.

![Compositionality Diagram]

**Figure 2.2**: compositional derivation of denotations
(16) Tina is not tall and thin.

(18) \[ \text{AP} \rightarrow \text{tall, thin, ...} \]
    \[ \text{AP} \rightarrow \text{AP and AP} \]
    \[ \text{AP} \rightarrow \text{not AP} \]

**Figure 2.3:** structural ambiguity
Structural ambiguity (2)

\[ \text{NOT}(A) = \overline{A} = E \setminus A = \text{the set of all the members of } E \text{ that are not in } A \]

**Figure 2.4:** Compositionality and ambiguity

(19)  
\begin{align*}
\text{a. } & \text{IS}(\text{tina}, \text{AND}(\text{NOT}(\text{tall}), \text{thin})) = 1 \iff \text{tina} \in \overline{\text{tall}} \cap \text{thin} \\
\text{b. } & \text{IS}(\text{tina}, \text{NOT}(\text{AND}(\text{tall}, \text{thin}))) = 1 \iff \text{tina} \in \overline{\text{tall}} \cap \text{thin}
\end{align*}

Note: Ambiguity vs. vagueness
Prerequisite: Naive Set Theory

- membership, equality, subset
- set specification
- empty set and set construction
- set union, intersection, complement, difference
- powersets
- ordered pairs, cartesian products
- relations, domain, range
- properties of relations: symmetry, transitivity...
- functions
- inverse functions, function composition
- injection, surjection, bijection

Partee, ter Meulen and Wall: *Mathematical Methods in Linguistics* Ch.1-3
Sets

- A set is an axiomatic/intuitive notion.
- Axiom system in Set Theory – Zermelo-Fränkel
- Naïve Set Theory
- Intuitively: a set is an abstract collection of objects, which are called the set members or elements.

\[ A = \{ \text{De Vries, Toledo, Winter} \} \]  
De Vries is element of \( A \)

\[ B = \{ \text{Chomsky, Montague} \} \]

\[ C = \{ A, B \} \]
\[ = \{ \{ \text{De Vries, Toledo, Winter} \}, \{ \text{Chomsky, Montague} \} \} \]

\[ D = \{ \text{Obama, Romney} \} \]  
De Vries is not element of \( C \)

\[ E = \{ C, D \} \]
\[ = \{ \{ \{ \text{De Vries, Toledo, Winter} \}, \{ \text{Chomsky, Montague} \} \}, \{ \text{Obama, Romney} \} \} \]
Elements and Subsets

\[ A = \{ \text{De Vries, Toledo, Winter} \} \]

*Toledo* is an element of *A*

\[ Toledo \in A \]

\[ B = \{ \text{De Vries, Toledo} \} \]

*B* is a *subset* of *A*: every element of *B* is an element of *A*

\[ B \subseteq A \]

**Note:** *Toledo* is *not* a subset of *A* and *B* is *not* an element of *A*.

The empty set \( \Phi \) has no elements; it is subset of every set.
Two sets are equal just in case they have the same elements. The order in which they are listed doesn’t matter.

\[ \{1, 2, 3\} = \{2, 1, 3\} = \{1, 3, 2, 3\} \]

\[ A = B \iff A \subseteq B \text{ and } B \subseteq A \]
Describing sets

Explicitly:
\[ A = \{ \text{De Vries, Toledo, Winter} \} \]
\[ B = \{1,2,3,4,5\} = \{5,4,3,2,1\} = \{1,2,2,3,4,5,5,2,4\} \]

In predicate notation:
\[ C = \{ x \in B \mid x \leq 4 \} \]
\[ C = \{1,2,3,4\} \]

By induction:
\[ 2 \in D; \]
\[ \text{if } x \in D \text{ then } x+2 \in D; \]
\[ \text{Nothing else is in } D. \]

\[ C = \{2,4,6,8,\ldots\} \quad C = \{x \in N \mid x \text{ is even}\} \]
Basic operations on Sets

$A \subseteq E$ and $B \subseteq E$

$A \cup B$, the **union set** of $A$ and $B$, is the subset of $E$ defined by:

$$A \cup B = \{ x \in E \mid x \in A \text{ or } x \in B \}$$

$A \cap B$, the **intersection set** of $A$ and $B$, is the subset of $E$ defined by:

$$A \cap B = \{ x \in E \mid x \in A \text{ and } x \in B \}$$

$A - B$, the **difference set** of $A$ and $B$, is the subset of $E$ defined by:

$$A - B = \{ x \in E \mid x \in A \text{ and } x \notin B \}$$

$\bar{A}$, the **complement set** of $A$, is the subset of $E$ defined by:

$$\bar{A} = E - A = \{ x \in E \mid x \notin A \}$$

Examples: ...

Prove: \[ \overline{X \cup Y} = \overline{X} \cap \overline{Y} \]
Simple proof - example

Prove: $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$

Proof:
Assume $x \in \overline{X \cup Y}$.
Thus, $x \notin X \cup Y$ (def. complement).
Now we establish $x \notin X$ and $x \notin Y$, because:
- If we had $x \in X$, then $x$ would be in $X \cup Y$ (def. union) – contradiction.
- If we had $x \in Y$, then $x$ would also be in $X \cup Y$ (def. union) – contradiction.
Therefore, $x \in \overline{X}$ and $x \in \overline{Y}$ (def. complement).
Thus, $x \in \overline{X} \cap \overline{Y}$ (def. intersection).

We have shown that for every $x \in \overline{X \cup Y}$ we have $x \in \overline{X} \cap \overline{Y}$.
We conclude $\overline{X \cup Y} \subseteq \overline{X} \cap \overline{Y}$.

Left to show: $\overline{X \cup Y} \supseteq \overline{X} \cap \overline{Y}$ – homework.
Tina is not tall and thin

**Figure 2.4:** compositionality and ambiguity

<table>
<thead>
<tr>
<th>A.</th>
<th>B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IS(tina, AND(\text{NOT}(tall), thin))</td>
<td>IS(tina, NOT(AND(tall, thin)))</td>
</tr>
<tr>
<td>tina IS AND(\text{NOT}(tall), thin)</td>
<td>tina IS NOT(AND(tall, thin))</td>
</tr>
<tr>
<td>NOT(tall) AND thin</td>
<td>NOT tall AND thin</td>
</tr>
</tbody>
</table>

a. \( IS(tina, \text{AND(\text{NOT}(tall), thin))) = 1 \quad \text{iff} \quad tina \in \overline{\text{tall}} \cap \text{thin} \)
b. \( IS(tina, \text{NOT(AND(tall, thin))) = 1 \quad \text{iff} \quad tina \in \overline{\text{tall}} \cap \text{thin} \)

Thus, under analysis A we have:

Tina is [[not tall] and thin] \( \Rightarrow \) Tina is not tall

Under analysis B we have:

Tina is [not [tall and thin]] \( \Rightarrow \) Tina is not tall

Thus, under analysis A we have: Tina is [[not tall] and thin] \( \Rightarrow \) Tina is not tall

Under analysis B we have: Tina is [not [tall and thin]] \( \Rightarrow \) Tina is not tall
Power Sets

\[ X \subseteq E: \]

The powerset of \( X \) is the set of subsets of \( X \):

\[ P(X) = \{ Y \subseteq E \mid Y \subseteq X \} \]

Let \( X \) be the set \{A, B, C, D, E\}. In our case, \( X \) has 32 subsets:

\[
X = \{A, B, C, D, E\}.
\]

\[
\emptyset, \\
\{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \\
\{A, B\}, \{A, C\}, \{A, D\}, \{A, E\}, \{B, C\}, \\
\{B, D\}, \{B, E\}, \{C, D\}, \{C, E\}, \{D, E\}, \\
\{A, B, C\}, \{A, B, D\}, \{A, B, E\}, \{A, C, D\}, \{A, C, E\}, \\
\{A, D, E\}, \{B, C, D\}, \{B, C, E\}, \{B, D, E\}, \{C, D, E\}, \\
\{A, B, C, D\}, \{A, B, C, E\}, \{A, B, D, E\}, \\
\{A, C, D, E\}, \{B, C, D, E\}, \\
\{A, B, C, D, E\}.
\]

Note that we started with the empty set \( \emptyset \).
We put all the subsets of \( X \) into one bigger set called \( P(X) \), the power set of \( X \).
For our \( X \), the set \( P(X) \) is one set with 32 elements.
Each element of \( P(X) \) is a subset of \( X \), and vice-versa.
A relation on a set $S$ is a set of ordered pairs from $S$.

Examples
First, suppose that $S$ is \{s, t, u, v\}.
Here are some examples of relations on $S$:
1. $R_1 = \{(s, t), (t, u), (v, v)\}$.
2. $R_2 = \emptyset$. This one has no pairs in it at all.
3. $R_3 = \{(s, s), (s, t), (s, u), (s, v), \ldots, (v, s), (v, t), (v, u), (v, v)\}$.
4. $R_4 = \{(s, s), (t, t), (u, u), (v, v), (s, u), (u, s)\}$.
$R_3$ contains all 16 ordered pairs, and it would usually be written as $S \times S$. 
Examples
Second, suppose that $S$ is the set of people in the room, call them $s_1, \ldots, s_n$.
$R_1 = \{(x, y) : x \text{ is strictly older than } y\}$.

Let $R_2$ be the set of pairs $(x, y)$ such that $x$ and $y$ have ever said hello to each other.
Let $R_3$ be the set of pairs $(x, y)$ such that $x$ admires $y$. 
Cartesian Products

\( X, Y \subseteq E: \)

The \textit{cartesian product of X and Y} is the set of pairs:

\[ X \times Y = \text{the set of pairs } (x, y) \text{ where } x \in X \text{ and } y \in Y \]

\[ \{1,2\} \times \{a,b,c\} = \{(1,a),(1,b),(1,c), (2,a),(2,b),(2,c), (3,a),(3,b),(3,c)\} \]

A relation on \( Y \) is a subset of the cartesian product \( Y \times Y \), which is also denoted \( Y^2 \).
Given two sets $A$ and $B$, a function from $A$ to $B$ is a rule or procedure that “inputs” elements of $A$ and “outputs” elements of $B$.

Notation:

$$ f : A \to B $$

Every element $a \in A$ gets “sent to” some element of $B$ which is called $f(a)$.

Functions are also called mappings.

$A$ is called the domain of $f$, and $B$ the codomain or range.
One-to-one and onto

\[ f : A \rightarrow B \] is one-to-one if different inputs go to different outputs.

\[ f : A \rightarrow B \] maps onto \( B \) if every element of \( B \) is the output of some element of \( A \), possibly more than one.

(Often people say “\( f \) is onto”, using a preposition as an adjective. Clearly they have no regard for traditional grammar!)

Fancier names: injective for one-to-one. surjective for “maps onto its codomain.”
We say that two sets $A$ and $B$ have the same size if there is a function $f : A \rightarrow B$ which is one-to-one and maps onto $B$.

These conditions can be expressed more formally:

If $a \neq a'$ in $A$, then $f(a) \neq f(a')$.

For every $b$, there is some $a \in A$ such that $f(a) = b$. 
Example

Here are two sets:

Numbers = \{0, 1, 2, 3, \ldots\}
Even = \{0, 2, 4, 6, \ldots\}

It might be surprising to think that these two sets have the same size, but this is what the definitions tell us.

To be sure, we need to find some function

\[ f : \text{Numbers} \to \text{Even} \]

which is one-to-one and onto.
We can take

\[ f(0) = 0 \\
\quad f(1) = 2 \\
\quad f(2) = 4 \\
\quad \vdots \\
\quad f(n) = 2n \\
\quad \vdots \]
Sets of functions

For any two sets $A$ and $B$, the set of all functions from $A$ to $B$ is denoted $B^A$ alternatively, $A \to B$.

Example:

$\{a, b, c\}^{\{1,2\}}$ = the following functions:

- $1 \mapsto a$, $2 \mapsto a$
- $1 \mapsto a$, $2 \mapsto b$
- $1 \mapsto a$, $2 \mapsto c$
- $1 \mapsto b$, $2 \mapsto a$
- $1 \mapsto b$, $2 \mapsto b$
- $1 \mapsto b$, $2 \mapsto c$
- $1 \mapsto c$, $2 \mapsto a$
- $1 \mapsto c$, $2 \mapsto b$
- $1 \mapsto c$, $2 \mapsto c$
Let $X$ be any set.

Every subset $A \subseteq X$ gives us a function $f_A : X \to \{0, 1\}$ called the characteristic function of $A$.

It is defined as follows:

$$f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

**Example:** $X = \{a, b, c, d\}$, $A = \{a, c\}$

Here are some examples of how this function $f_A$ works:

- $f_A(a) = 1$.
- $f_A(b) = 0$.
- $f_A(c) = 1$.
- $f_A(d) = 0$. 
Currying

F: \((M \times W) \rightarrow [0,1]\)
F gives any pair of man and woman \((m,w)\) a score \(F(m,w)\)
indicating matching

G: \(M \rightarrow (W \rightarrow [0,1]))\)
G gives any man \(m\) a function \(G(m)\) mapping any woman \(w\) to a
score \((G(m))(w)\).

Thus, we can define: \((G(m))(w) = F(m,w)\)
We say that G is the Curried version of F, and that F is the
deCurried version of G.